ADAPTIVE FORECASTING OF WAVE NON-PERIODIC TIME SERIES

Leonid Lyubchyk, Galina Grinberg ∗, Arkady Rivtis ♦

∗ National Technical University
"Kharkiv Polytechnic Institute",
21 Frunze str., Kharkiv, 61002, UKRAINE
E-mail: Lyubchik@kpi.kharkov.ua

♦ Tutoring Center of Mathematics & Sciences,
1101 Gail Dr., Buffalo Grove, IL 60089, USA
E-mail: Rivtisa@aol.com

Abstract. The identification and forecasting problem of non-periodic financial time series with wave structure is considered. The adaptive forecasting method is proposed using the special auto-regression representation of wave series and both frequencies and amplitudes of partial harmonics identification. Model simulation results are presented and the results of real problem solution of stock prices forecasting and trading decision support are also considered.

Key words: identification, financial engineering, forecasting, signal processing, time series.

1 Introduction

The necessity of complex time series forecasting takes place in many econometrics problems [1]. The efficiency and accuracy of forecasting essentially depends from the adequacy of time series model. The popular forecasting methods usually use the simple models like "trend + noise" or ARMA models in combination with recurrent parameters identification algorithms [2,3]. However, in practice of econometric forecasting the real time series have a more complex structure, such as non-periodic oscillating time series. Even the simple superposition of a number of harmonics with aliquant frequencies leads for the time series structure mentioned above. Such a functions, so calls *wave time series*, may be sufficiently used as mathematical models of financial time series [4].

The identification problem of such time series is considerably simple in the case, when the wave component is the superposition of a number of harmonics with known frequencies and phases, at that the unknown amplitudes identification may be performed by simple algorithms like recurrent least square method. The problem has become more complicated when both frequencies and phases are arbitrary unknown and moreover changing in time. In general case such time series are non-periodic and unknown frequencies extraction by means of the Discrete Fourier Transform (DFT) method [5] is actually impossible.

In this paper an approach for wave time series forecasting is proposed based on a special assignment of wave component auto-regression model as a superposition of harmonics with tuning frequencies. In such a case the suitable identification algorithm also ensures non-stationary frequencies tracking. The proposed method based on the structural modeling approach [6,7] is seemed to be very useful for seasonal and oscillating economic and financial time series.

2 Wave time series model identification

Consider the trend-seasonal time series model

$$Y_k = \sum_{i=0}^{n} D_i k^i + \sum_{j=0}^{m-1} \left[A_j \cos \omega_j k + B_j \sin \omega_j k \right] + \xi_k , \qquad (1)$$

where Y_k - the time series value at instant k, n - the degree of polynomial component, m - number of harmonics with frequencies $0 < \omega_j = 2\pi f_j T_0 < \pi$, T_0 - sampling period, ξ_k random zero-mean measurement noise.

The necessary step of time series forecasting is the identification of mathematical model parameters D_i, A_j, B_j, ω_j. Because the identification of trend and wave components performs by different methods, at first it is necessary to divide the components of time series. At that the trend component may be extract by the following two methods:

A) <u>Discrete smoothing method.</u>
In such a case trend is extracted by means of law-pass filter realized by exponential smoothing algorithm. Let $n = 0$, i.e. trend is a constant bias D_0. Then using the exponential smoothing procedure

$$\overline{Y}_k^s = \alpha_s \overline{Y}_{k-1}^s + (1-\alpha_s) Y_k, \quad 0 < \alpha_s < 1, \tag{2}$$

the initial time series (1) can be divided to the slow component (trend) estimation $\overline{Y}_k^s$ and the fast one $\widetilde{Y}_k = Y_k - \overline{Y}_k^s$, which is the linear transformation of the wave component distorted by the random noise $\widetilde{\xi}_k$,

$$\widetilde{\xi}_k = \xi_k - \overline{\xi}_k^s, \quad \overline{\xi}_k^s = \alpha_s \overline{\xi}_{k-1}^s + (1-\alpha_s)\xi_k . \tag{3}$$

It is evident that the smoothing transformation doesn't change the wave component spectrum. In general case $n > 0$ it is possible to apply the high order exponential smoothing.

B) <u>Discrete differentiation method</u>

In such a case the trend extraction is performed by discrete differentiation of time series (1), which is previously smoothed by discrete wide-band filter in the purpose of noise reduction:

$$\begin{aligned} &\widetilde{Y}_k^d = \alpha_d \widetilde{Y}_{k-1}^d + (1-\alpha_d) Y_k, \quad 0 < \alpha_d < 1, \\ &V_k = \widetilde{Y}_k^d - \widetilde{Y}_{k-1}^d = (1-\alpha_d)\left(Y_k - \widetilde{Y}_{k-1}^d\right) \end{aligned} \tag{4}$$

As a result the first difference sequence V_k has the structure of wave component distorted by the equivalent noise

$$\nu_k = \frac{1-\alpha_d}{\alpha_d}\left(\xi_k - \widetilde{\xi}_k^d\right), \quad \widetilde{\xi}_k^d = \alpha_d \widetilde{\xi}_{k-1}^d + (1-\alpha_d)\xi_k . \tag{5}$$

At that both methods lead to the identification problem of wave time series

$$y_k = \sum_{j=0}^{m-1} \left[a_j \cos \omega_j k + b_j \sin \omega_j k\right] + \zeta_k , \tag{6}$$

where $y_k = \widetilde{Y}_k$, $\zeta_k = \widetilde{\xi}_k$ in case (a) and $y_k = V_k$, $\zeta_k = \nu_k$ in case (B). Using z- transformation, the model (6) may be presented in the form:

$$\prod_{j=0}^{m-1} \left[1 - 2\cos \omega_j z^{-1} + z^{-2}\right] y_k = \zeta_k . \tag{7}$$

Realizing the inverse transition in time domain the equation (6) may be represent in the linear auto-regression form:

$$y_k = \sum_{j=0}^{m-1} \beta_j \left(y_{k+j-m} + y_{k-j-m} \right) - y_{k-2m} + \zeta_k = \beta^{\mathrm{T}} y(k,m) - y_{k-2m} + \zeta_k, \quad (8)$$

where $y(k,m) = (2y_{k-m},\ \ y_{k-m+1} + y_{k-m-1}, \ldots, y_{k-1} + y_{k-2m+1})^{\mathrm{T}}$ is the time series "prehistory" vector, $\beta^{\mathrm{T}} = (\beta_0, \beta_1, ..., \beta_{m-1})$ - model parameters.

Using the quadratic identification criterion

$$J = \sum_{k=2m}^{N-1} \left[y_k + y_{k-2m} - \beta^{\mathrm{T}} y(k,m) \right]^2 \quad (9)$$

one can obtain the recurrent algorithm of the model identification:

$$\begin{aligned} \hat{\beta}_k &= \hat{\beta}_{k-1} + [y_k + y_{k-2m} - \beta_{\mathrm{k-1}}^{\mathrm{T}} y(k,m)] y(k,m) r_k^{-1}, \\ r_k &= \gamma_k r_{k-1} + \| y(k,m) \|^2, \quad 0 < \gamma_k < 1, \end{aligned} \quad (10)$$

where tuning parameter γ may be used for trade-off adjusting between tracking and flittering properties of the algorithm (10). Using the non-parametric criterion of time series property changing, the following tuning algorithm may be used:

$$\begin{aligned} \gamma_k &= \gamma_{k-1} + \Delta\gamma, \quad \left| \sum_{i=k-q}^{k} \mathrm{sign}(y_i - \hat{y}_i) \right| \le \delta, \\ \gamma_k &= \gamma_{k-1} - \Delta\gamma, \quad \left| \sum_{i=k-q}^{k} \mathrm{sign}(y_i - \hat{y}_i) \right| > \delta, \end{aligned} \quad (11)$$

where q is a memory depth, δ и $\Delta\gamma$ - parameters,

$$\hat{y}_i = \hat{\beta}_{i-1}^{\mathrm{T}} y(i,m) + y_{i-2m} \quad (12)$$

is an optimal one step prediction of time series obtained by current estimates.

Unknown frequencies ω_j are connected with the estimated parameters β_j by the equation

$$\beta_0 + \sum_{j=1}^{m-1} \beta_j \cos j\omega = \cos m\omega . \tag{13}$$

Taking into account that

$$\cos m\omega = \cos^m \omega - C_m^2 \cos^{m-2} \omega \sin^2 \omega + C_m^4 \cos^{m-4} \omega \sin^4 \omega + \ldots, \tag{14}$$

frequencies ω_j may be determined as a roots of power polynomial from the argument $\cos\omega$.

Similarly at any instant k the estimations of wave component harmonics amplitudes $\Theta = (a_0, a_1, \ldots, a_{m-1}, b_0, b_1, \ldots, b_{m-1})^{\mathrm{T}}$ may be obtained by the quadratic identification criterion minimization

$$J_\Theta = \left\| Y(k,m) - \Phi(k,m)\Theta \right\|^2, \tag{15}$$

where the extended vector $Y(k,m)$ and matrix $\Phi(k,m)$ are defined as

$$Y(k,m) = \begin{bmatrix} y_{2m} \\ y_{2m+1} \\ \vdots \\ y_k \end{bmatrix}, \ \Phi(k,m) = \begin{bmatrix} \cos 2m\hat{\omega}_0 & \ldots & \cos k\hat{\omega}_0 \\ \vdots & \ldots & \vdots \\ \cos 2m\hat{\omega}_{m-1} & \ldots & \cos k\hat{\omega}_{m-1} \\ \sin 2m\hat{\omega}_0 & \ldots & \sin k\hat{\omega}_0 \\ \vdots & \ldots & \vdots \\ \sin 2m\hat{\omega}_{m-1} & \ldots & \sin k\hat{\omega}_{m-1} \end{bmatrix}$$

and $\hat{\omega}_j$, $j = \overline{0, m-1}$ are the frequencies estimations. As a result in accordance with general Least Squares Method

$$\hat{\Theta} = \left(\Phi\Phi^{\mathrm{T}}\right)^{-1} \Phi Y . \tag{16}$$

3 Optimal time series forecasting

Optimal forecast of time series wave component for p steps $\hat{y}_{k+p}$ may be obtained using the one step forecast (12) in the form:

$$\hat{y}_{k+p} = \hat{\beta}_k^{\mathrm{T}} \hat{y}(k+p, m) - y_{k+p-2m}, \ p \ge 1, \tag{17}$$

where elements y_i, $k \le i \le k+p-1$ in vector $\hat{y}(k+p,m)$ are replaced by their forecasting values calculated in accordance with (17).

The obtained expressions may be used for initial time series (1) short-term forecasting in accordance with accepted method of components separation and structural wave analysis. For method (A) the forecasting formula is

$$\hat{Y}_{k+p} = \overline{Y}^{s}_{k+p} + \tilde{Y}_{k+p}, \tag{18}$$

where $\overline{Y}^{s}_{k+p}$ is a trend component and $\tilde{Y}_{k+p}$ is a wave component forecast.

For the trend component forecasting it is expediently to use the exponential smoothing method. Then

$$\overline{Y}^{s}_{k+p} = \sum_{i=0}^{n} \overline{D}_{ik} p^{i}, \tag{19}$$

where coefficients $\overline{D}_{ik}\left(S_k^0,\ldots,S_k^n\right)$ are expressed using the exponential averages S_k^l, $l=\overline{0,n}$:

$$S_k^l = \alpha_p S_{k-1}^l + \left(1-\alpha_p\right) S_k^{l-1},\ S_k^{-1} = \overline{Y}_k. \tag{20}$$

The wave component forecast is carried out using the proposed technique.

For method (B) realizations taking into account an evident relation between time series component

$$Y_k - \tilde{Y}_k = \alpha_d \left(1-\alpha_d\right)^{-1} V_k \tag{21}$$

the forecast formula may be represent as:

$$\hat{Y}_{k+p} = \tilde{Y}_k^d + \sum_{i=1}^{p} \hat{V}_{k+i} + \frac{\alpha_d}{1-\alpha_d} \hat{V}_{k+p}, \tag{22}$$

In such a case time series first difference forecast is carried out using the proposed frequency estimation method. As a result the wave non-periodic time series forecasting may be performed by simple recurrent algorithm.

4 Simulation results

Fs an example consider the simulation results for proposed forecasting algorithm. The initial time series is chosen as the superposition of three harmonics with frequencies $\omega = [2.51 \quad 1.14 \quad 0.50]$ and amplitudes $A = [0.8 \quad 1.5 \quad 1]$. The measurements are distorted by the random noise with uniform distribution in interval ± 0.5. The simulation results (Fig. 1) illustrated that DFT extracts a fictitious harmonics whereas the proposed method ensures the stable estimation of both frequencies and amplitudes of time series components and ensure the high quality forecasting.

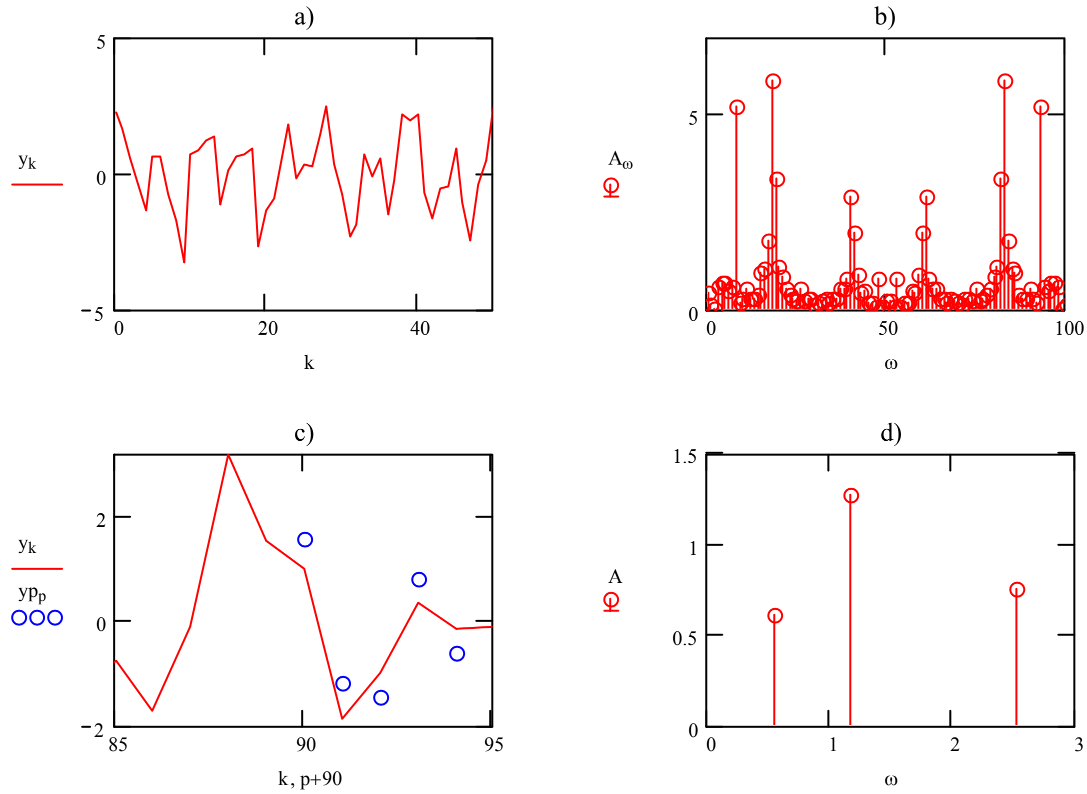

Fig.1. Forecasting algorithm simulation
a) initial time series; b) DFT of time series;
c) time series forecasting; d) estimated frequencies of partial harmonics.

5 Example: stock prices forecasting

The proposed method is applied for the problem of stock prices forecasting and buy/sell decision support [8]. The method is based on the idea of harmonic structure analysis of stock prices time series. Many samples of stock prices have a wave (non-periodic oscillating) structure so can be represent as a combination of the number of harmonics with unknown and changing frequencies and amplitudes. The information about this harmonics may be very useful both for time series forecast and decision support.

The peculiarities of the method are the initial time-series decomposition on the slow (trend) and fast (oscillatory) components with the help of digital filters. The special adaptive technique is used for model updating with the combination of detection of the days when the prices time series change its properties.

The proposed procedure of data processing and decision includes: identification of harmonic models using accumulated data (amplitudes and frequencies as well as necessary number of harmonics estimation) and short-term forecasting of stock prices and decision function construction in order to obtain the buy/sell decision recommendation in current day:

- *buy*, if the price is near the local minimum in current day and predicted price increases;
- *sell,* if the price is near the local maximum and the predicted price decreases;
- *hold* , in over cases.

Computer simulation has been fulfilled in order to evaluate the performance of the proposed method and algorithms. The real data of stock prices (Fig.2) has been used. The time series processed step-by-step (one step is one day), moreover in any current day only the previous data are assumed to be known so the actual behavior of stock trader is simulated.

For each step the following calculations are performed:

- the initial time series is separated into the slow (trend) and fast (wave) components by means of digital filtering algorithms (Fig. 3,5,7).
- the harmonic components of both trend and wave terms as well as first difference are estimated (Fig. 4,6,8) using the developed techniques. The four harmonics model is used. Moreover, the number of efficient harmonics is determined.
-the short term (5 days ahead) forecast based on the estimated harmonic model (Fig.9) is calculated (the solid line – real data, dotted line – forecast). In Fig. 10 the same curves are presented in enlarges scale.
- using the forecasted data, the decision rule is created. The segments of time series immediately proceeding to the current step and forecast are approximated by the second-order curves and the local extremum are calculated. The decisions "buy" or "sell" is accepted if the estimated local minimum or maximum is located near the current day respectively. The decision "hold" is accepted in any over cases. The fragment of decision sequence is presented at Fig. 11.
- passing on to the next steps the actual income or losses are calculated using the accepted decision and real prices. In Fig. 12 the accumulated relative income

from the first to current day is presented, i.e. the actual capital increment per one stock attained by the stock trading using the proposed forecasting method and decision rule.

The simulation results demonstrate the stable grows of the income even in the case when the trend of stock prices has the tendency to the decreasing. Of course, it takes place only in a long period of trading (one year or more) when the frequency of a right decision exceeded the faulty one.

Conclusion

The proposed technique ensures the forecasting of financial non-periodic time series with wave structure. It can be treated as the development of structural approach for spectral analysis [7] and may be used in non-stationary case, when the time series model structure and parameters are changed in time. In such a way the critical point is the determining the optimal number of harmonics in the predictive model of wave component. Such a choice may be done using a multi-model approach, at that the adaptive algorithm of high level may adjust the model structure [9].

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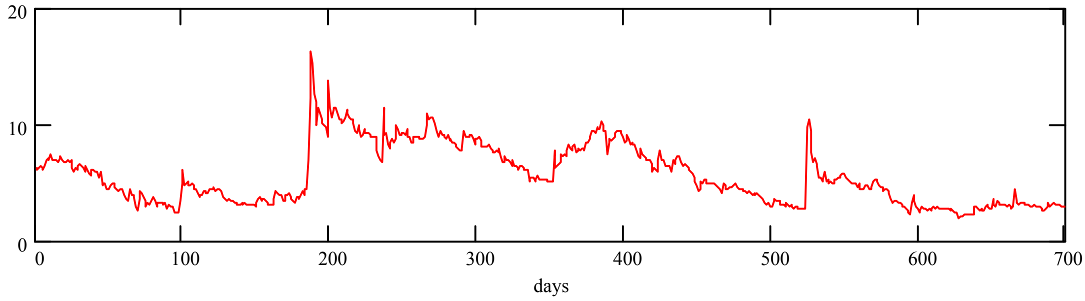

Fig 2. Stock prices

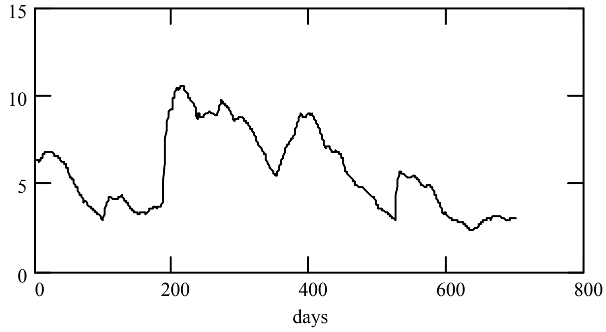

Fig 3. Trend (slow) component

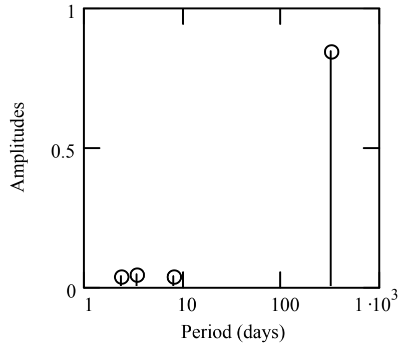

Fig 4. Trend's harmonics

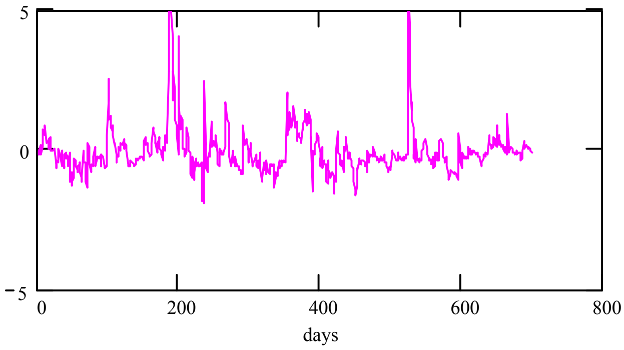

Fig 5. Wave (fast) component

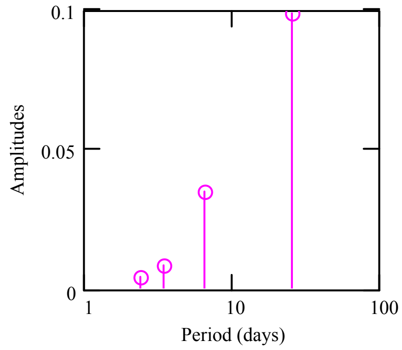

Fig 6. Wave's harmonics

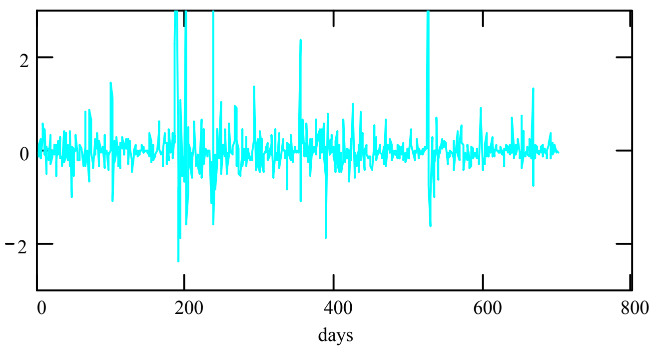

Fig 7. First difference

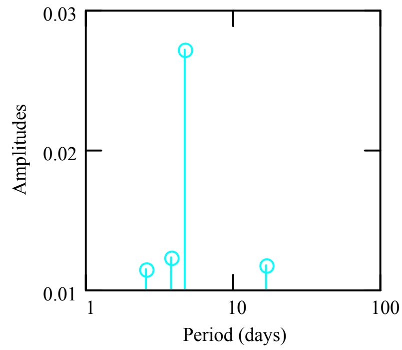

Fig 8. Difference's harmonics

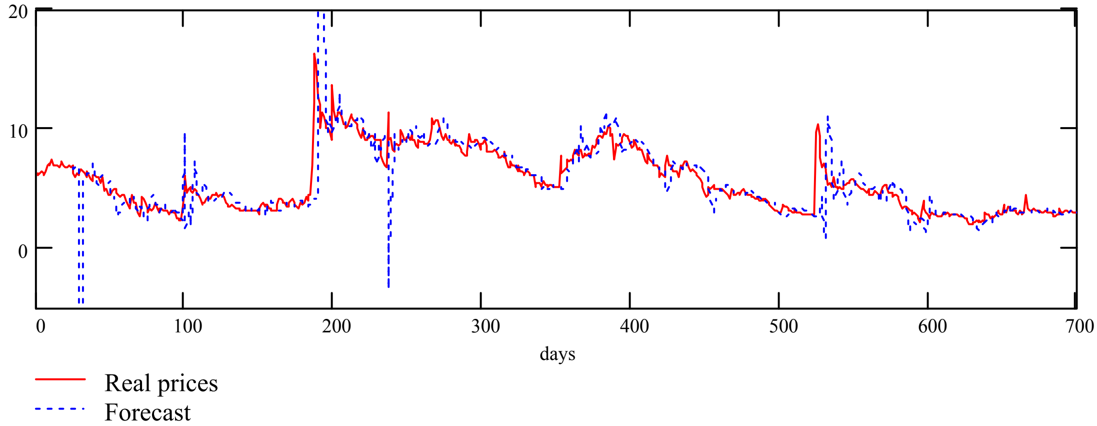

Fig 9. Five step ahead forecast

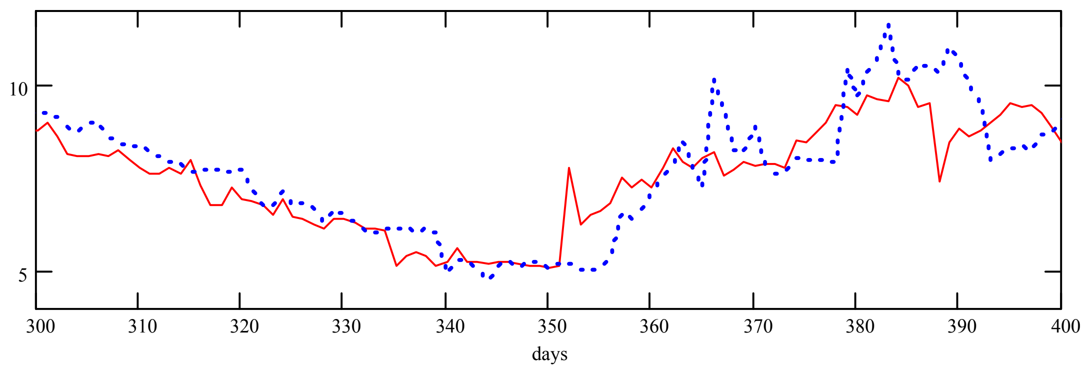

Fig 10. Forecast (zoom)

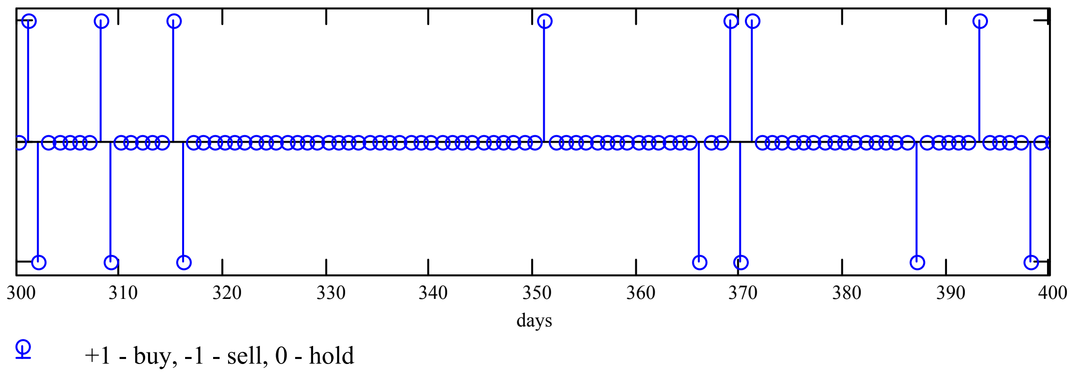

Fig 11. Buy/sell decision

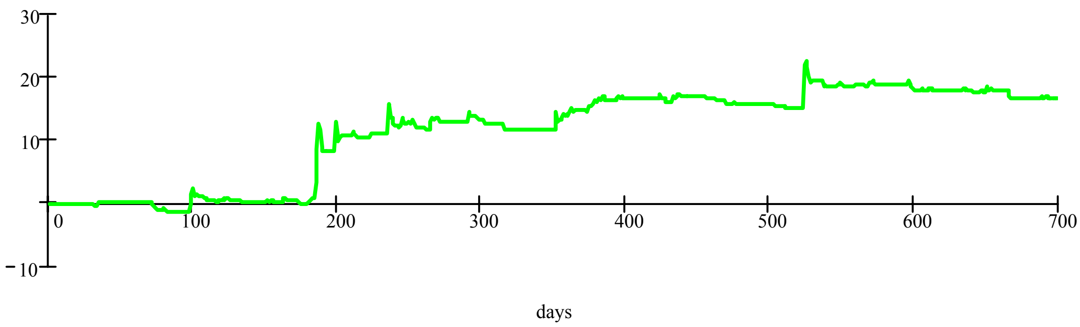

Fig 12. Current profit